Improving the Numerical Efficiency of Generalized Multipole Technique by Non-redundant Multipole Choices

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Abstract: The Generalized Multipole Technique, due to its flexibility, is used in a variety of cases for the analysis of electromagnetic structures. This method is generally based on a multiple multipole expansion and a point matching technique. The numerical conditioning of the matrices involved in this analysis is strongly dependent on the matching point and the multipole distribution. In this contribution, we use the well-known Singular Value Decomposition to investigate systematically the numerical conditioning of these matrices. We suggest a method to improve the conditioning of the procedure in case of an ill-conditioned system and we validate it by evaluating the error in field matching and the far field radiation pattern in case of a radiating elliptical aperture.

Keywords: Generalized multipole technique, Near field analysis, Far field analysis, Numerical analysis, Scattering at apertures.

1. Introduction

In the conventional approach for evaluating radiation field at apertures, often it is assumed that the aperture is mounted on an infinite metallic flange plane[1].Unfortunately, with this approach, the radiation pattern is obtained in a limited accuracy zone in the far field region and the back-scattered field remains completely unknown. Moreover, the real three-dimensional geometry of the conducting structure containing the radiating aperture cannot be taken into account in the radiated field evaluation.

The use of Generalized Multipole Technique (GMT) together with the Point Matching Technique allows us, by imposing proper boundary condition, to accurately model the radiating aperture together with the real three-dimensional metallic structure on which the aperture is mounted. In this method the external field is represented in terms of the elementary fields radiated by finite sets of multipoles, which are distributed within the radiating object. Here, the aperture field is considered to be known. As an example, it can be obtained by applying the Generalized-Multipole-Technique – Mode-matching-Technique as in [2]-[3] where the aperture field is expressed in terms of the aperture eigen-modes.

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A key-point in this method is the distribution of the multipoles inside the structure under examination, e.g. a radiating object, and the distribution of the matching points on its conducting surface. The conditioning of this problem is directly related with the distribution of the multipoles and the matching points. The field generated by a distant multipole at neighboring matching points or the field produced by two very closely located multipole at a certain matching point are almost identical. This leads to linear dependency in the system of equation resulting severe ill conditioning. Though there are some guidelines to avoid the ill conditioning in the literature[4], but no systematic method to improve the numerical conditioning is suggested in case of a ill conditioned system.

A reliable and robust means to detect the linear dependency and conditioning of a matrix is the Singular Value Decomposition (SVD). We use SVD to estimate the liner dependency and conditioning of the matrices. We calculate a quantity, which is equivalent to the angle between two vectors in the Hilbert space and use this angle value to choose the multipoles that are needed in the analysis and eliminate the unwanted ones. This procedure considerably improves the numerical conditioning of the matrices involved hence improving the numerical efficiency of this method.

2. Theory

A. GMT method:

With reference to Fig.1, the electromagnetic field at an external point ‘r = (x,y,z)’ can be expressed by the multiple multipole expansion as:

$$E^{(ext)}(r) = \sum_k \sum_l \mathbf{Q_{kl}} E_l^{(mult)}(r - r_k) \quad (1)$$

$$H^{(ext)}(r) = \sum_k \sum_l \mathbf{Q_{kl}} H_l^{(mult)}(r - r_k) \quad (2)$$

Where, $E_l^{(mult)}$ and $H_l^{(mult)}$ are the multipole fields radiated by an l-th order multipole, r_k denotes the location of corresponding multipole and $\mathbf{Q_{kl}}$ denotes the unknown weight factors for the individual multipole contribution. In order to find out the unknown weight factor **Q**’s, the boundary conditions are imposed at the ‘m’ matching points on the aperture surface (S_a) and metallic surface (S_c) of the antenna. If we consider total ‘n’ number of multipoles, the tangential electric and magnetic and field matching on S_c and S_a leads us to the following set of equations,

$$\mathbf{v} = [C] \cdot \mathbf{Q} = [D] \cdot \mathbf{V} \quad (3)$$

Where, **v** is the vector containing the tangential electric fields on the matching points (S_a and S_c), each column of the m×n matrix [*C*] contains the tangential electric field components of a certain multipole

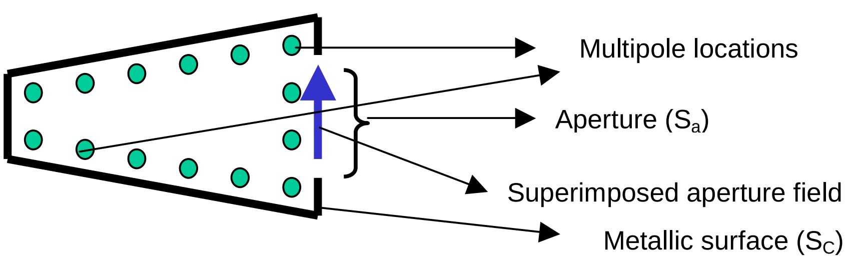

Fig.1. Schematic diagram showing the aperture antenna and the multipole locations

at the matching points, the i-th column of $[D]$ matrix contains the electric field components of the i-th aperture waveguide mode on field matching points (this is zero for field matching points in S_c) and the vector ***V*** contains the aperture modal voltages. From (3) one can obtain the unknown **Q**'s as

$$\mathbf{Q} = \left([C]^H \cdot [C]\right)^{-1} \cdot [C]^H \cdot [D] \cdot \mathbf{V} \tag{4}$$

Eq. (4) allows us to evaluate **Q** vector in terms of the known modal expansion coefficients **V**. Inserting **Q**'s coefficients into (1) we obtain the electric field on the surface S_a, S_c and on the external space. If the problem is well conditioned the field on the surfaces should agree with the imposed boundary conditions (apart from a small error) and then the electric external field and the relevant magnetic field (1)-(2) represent our solution. From (4) we notice that in order to determine the unknown expansion coefficients (**Q**) we need to invert the product of the matrices $[C]^H \cdot [C]$. As the C-matrix contains the multipole fields at the matching points, this matrix is our main point of interest. Here it may be noted that we deal with an over-determined system of equations, having the C-matrix as a rectangular matrix with complex elements. A detailed description of the whole method can be found in [2]-[3].

B. Singular value decomposition:

We can decompose the m×n complex matrix 'C' as

$$C = U \cdot \Sigma \cdot V^H \tag{5}$$

Where U and V are unitary matrices and Σ = diag ($\sigma_1, \sigma_2, \sigma_3 \ldots\ldots \sigma_r$), r = min(m,n) with $\sigma_1, \sigma_2, \sigma_3 \ldots\ldots \sigma_r \geq 0$ and V^H denotes the Hermitian conjugate of *V*. This decomposition is called the singular value decomposition (SVD) of the matrix 'C', and $\sigma_1, \sigma_2, \ldots$ are called singular values of 'C' [6]. The condition number of this matrix can be defined as,

$$Cond\ (C) = \|C\|_2 \cdot \left\|C^{-1}\right\|_2 = \frac{\sigma_{\max}}{\sigma_{\min}} \tag{6}$$

Where $\|C\|_2$ denotes the 2nd norm of the matrix 'C'. Large condition number of a matrix depicts poor conditioning with respect to inversion. From (6) it is clear that if some of the singular values of a matrix are zero or approach zero then the condition number of the matrix approaches infinite and the matrix is ill conditioned for inversion and is denoted as column rank deficit. The number of singular values that are very close to zero (or below a certain numeric value which can be taken as precision) are counted to have the number of linearly dependent columns in the matrix.

C. Inner product and angle between vectors:

The angle between two n-dimensional vectors (u and v) in real inner product space is defined as

$$\theta = \cos^{-1} \frac{\langle u.v \rangle}{\|u\|_2 \cdot \|v\|_2} \tag{7}$$

Where $\langle u.v \rangle$ denotes the inner product between the vectors u and v. When two vectors have zero or very little angle between them then they are parallel or very nearly parallel, respectively, which means they are linearly dependent or nearly linearly dependent. We use the same approach to detect the linear dependency in the GMT method. As we have all the elements of the 'C' matrix as complex elements, we calculate the angle according to the following formula for each columns of the 'C' matrix.

$$A_{ij} = \cos^{-1}\left(\frac{\left|C_j^* \cdot C_i\right|}{\left\|C_i\right\|_2 \cdot \left\|C_j\right\|_2}\right) \tag{8}$$

Where C_i denotes the *i*-th column of the 'C'- matrix. Both the subscripts 'i' and 'j' run from 1 to 'n'. Here, we need to calculate only the elements in the upper or lower triangle, as $A_{ij} = A_{ji}$. Two columns having smaller 'A'-value contain less information with respect to each other than two columns having a larger 'A' value. We sort out the columns having lowest 'A' values between them and delete one of them. Deleting a certain column is equivalent to delete one multipole as each of the columns represents the field components due to a particular multipole.

3. Simulated results

We consider the case of an elliptical horn antenna with an elliptical aperture as the radiating aperture. To simplify the calculations, we consider em symmetry (i.e. electric wall at xz plane and magnetic wall at yz plane) and assume that only a fundamental mode TE^e_{11} is excited at the aperture. We have simulated a horn with an elliptical aperture of semi major axis = 45.15 *mm*, semi minor axis = 33.85 *mm* and with a circular flange of radius = 70 *mm* [5]. The diagram of the horn with the aperture is shown in Fig. 2. The multipole locations inside the antenna and the matching points on the metallic surface and aperture can be seen in Fig 3.

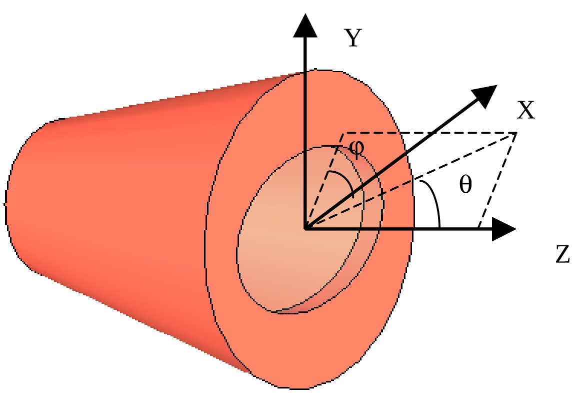

Fig. 2. Diagram of the simulated horn and the relevant co ordinate system.

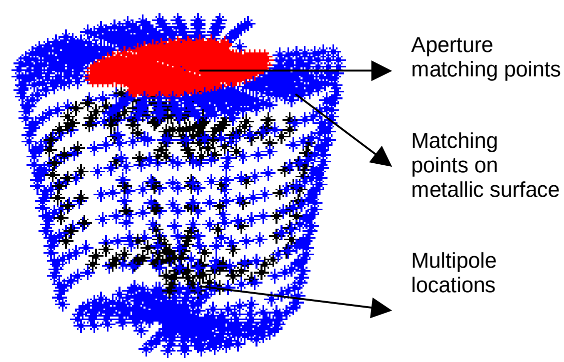

Fig. 3. The matching point distribution and multipole locations.

In the simulation, the following parameters have been used for point matching and multipole distribution:

Aperture matching points = 625
Matching points on the metallic surface =1557
Number of multipole locations =201
Total number of multipoles in all locations = 1206.

In order to know weather the 'C' matrix is well conditioned or not, we decompose it to obtain the singular values. In Fig. 4 the singular values are plotted against their number. We may note that the number of singular values is equal to the number of the columns (n) of the 'C' matrix. From the plot, it can be noticed that a large number of singular values lie between one and zero depicting the linear dependencies in the system of equation and severe ill conditioning of the ‘C’ matrix. The condition number of ‘C’ matrix in this case is 4.12×10^{09} with the highest singular value being 274.66 and the lowest one being 6.65×10^{-8}.

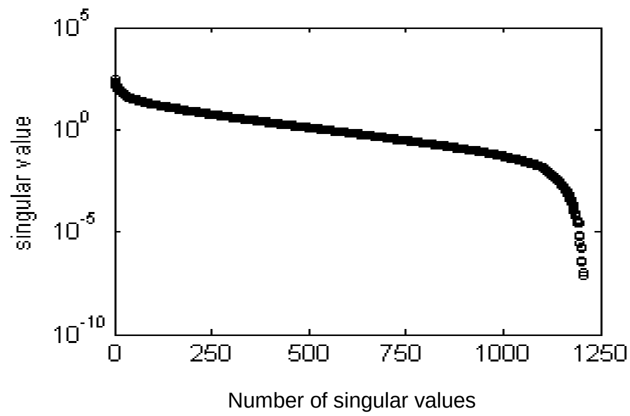

Fig 4: Singular value spectrum of ‘C’ matrix before column reduction.

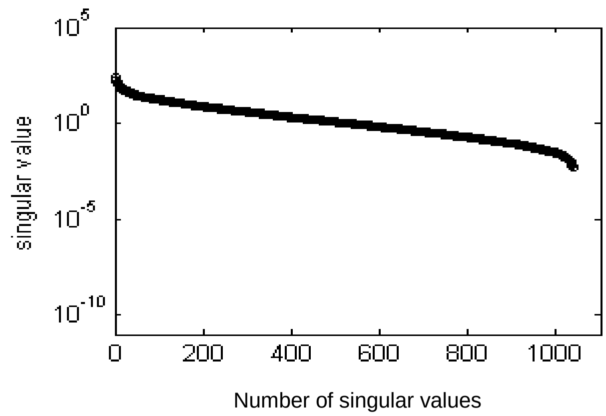

Fig 5: Singular value spectrum of ‘C’ matrix after column reduction.

Now the proposed method to improve the conditioning is applied on the ‘C’ matrix. After obtaining the ‘A’ matrix, 169 columns have been identified having the lowest angle values. The singular value spectrum of the ‘C’ matrix after eliminating these columns is shown in fig 5. For the new ‘C’ matrix the highest and lowest singular values are 227.55 and 0.0047 respectively, the condition number being 4.8×10^{4}.

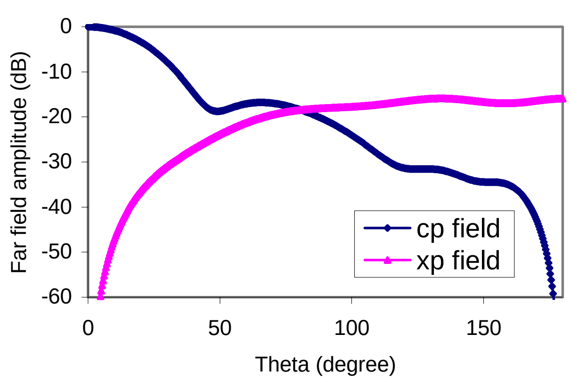

Fig 6: Co-polarized (cp) and cross-polarized (xp) field obtained for the elliptical aperture with $\varphi = 45°$ and excited with TE^{e}_{11} mode, before the improvement of the conditioning.

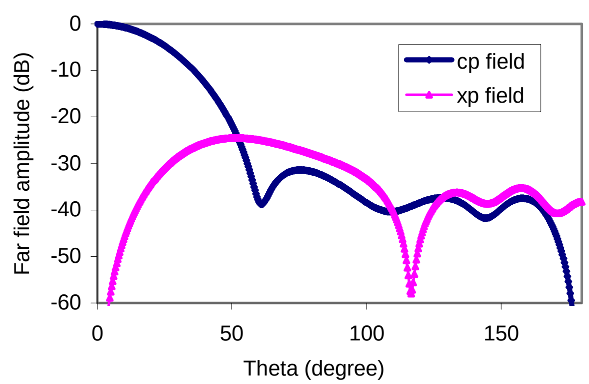

Fig 7: As in figure 6 but after the improvement of the conditioning.

In Fig.6 and Fig.7 the far field plots for the aperture are shown before and after column reduction of ‘C’ matrix respectively. It can be easily seen that co and cross-polarized fields obtained before the column reduction is wrong. The corresponding angles (θ,φ) have been shown in Fig.2.

In Fig.8 the error in field matching at the antenna surface and aperture after the column reduction is shown. Here we have plotted the difference between the ideal field (representing the boundary condition in our problem) and the obtained field at the antenna surface and the aperture at some arbitrary points other than the matching points. It is clear from the figure that overall quality of the field matching is very good. It is worthy to note that the maximum normalized error at the aperture is 0.0096 and the maximum difference in e-field on the metallic surface is 0.105.

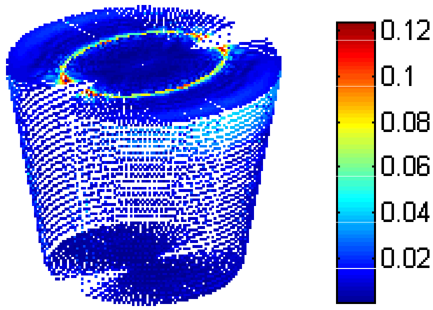

Fig 8: The error in field matching at the antenna surface and aperture. The color bar represents the value of the error.

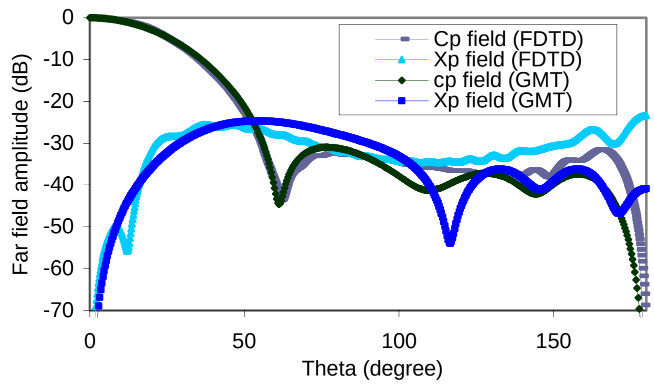

Fig 9: Comparison between the field pattern obtained by GMT (after column reduction) and FDTD (MWST).

In Fig.9 the comparison between the co and cross-polarized far field pattern of the same aperture obtained by our method (after improving the numerical conditioning) and the Finite Difference Time domain (FDTD) method is shown. For FDTD simulation, we used the Microwave studio (MWST) software and absorbing boundary condition has been applied to obtain the back-scattered field.

4. Conclusions

We have analyzed the issue of numerical conditioning in the Generalized Multipole Method applied to characterize the radiating aperture. The singular value decomposition is used to obtain the number of linearly dependent columns of the matrix. It has been shown that the idea of calculating the angle between two vectors in the inner product space can be applied successfully to identify the linear dependency in the involving system of equations. As this method is simple and general, so it can be a useful tool in automatic multipole setting in GMT and can also be used to remove ill conditioning in GMT applied to the complicated scattering structures.

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